

## Lecture VIII:

# International Financial Integration, Market Incompleteness, and Sovereign Risk

# International Risk Sharing

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- If international financial markets are complete markets in the Arrow-Debreu sense, domestic residents can be insured against all types of risk.
- The optimization problem becomes then analogous to that of dynamic optimization under certainty.
- Thus, the complete market assumption yields clear, albeit very strong, predictions on a host of issues.
- Here we will focus on predictions regarding international consumption correlations.

# International Risk Sharing

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- A key implication of complete financial markets at the international level is that all individuals in home and foreign countries can equate their marginal rates of substitution between current consumption and state-contingent future consumption to the same state-contingent security prices.
- Start with the domestic resident having access to a full set of Arrow-Debreu securities so that:

$$\underbrace{p_t(s_{t+1}) \frac{u'(C_t)}{P_t}}_{\text{Loss of utility of buying one unit of Arrow security}} = \underbrace{\frac{\pi(s_{t+1}) \beta u'(C_{t+1})}{P_{t+1}}}_{\text{Marginal utility pay-off upon realization of } s(t+1)} \quad (8.1)$$

Loss of utility of buying one unit of Arrow security      Marginal utility pay-off upon realization of  $s(t+1)$

# International Risk Sharing

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- Call  $Q_{t,t+1} = p_t(s_{t+1}) / \pi(s_{t+1})$  the stochastic discount factor, then:

$$\beta \frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} = \frac{p_t(s_{t+1})}{\pi(s_{t+1})} = Q_{t,t+1}$$

Under CARA utility, it becomes:

$$\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} = Q_{t,t+1} \quad (8.2)$$

As the foreigner has access to the same security with the same pay-off in domestic currency, the analogous condition will hold:

# International Risk Sharing

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- the stochastic discount factor, then:

$$\beta \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*} \frac{\varepsilon_t}{\varepsilon_{t+1}} = Q_{t,t+1} \quad (8.3)$$

where the exchange rate term converts the price index of the foreign basket to that of the home country unit.

Combining (8.2) with (8.3) yields

$$\left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} = \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*} \frac{\varepsilon_t}{\varepsilon_{t+1}}$$

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Re-arranging yields:

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} = \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{\varepsilon_t P_t^* / P_t}{\varepsilon_{t+1} P_{t+1}^* / P_{t+1}} = \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{RER_t}{RER_{t+1}} \quad (8.4)$$

Taking logs and first differencing then yields:

$$\begin{aligned} -\sigma[\ln c_{t+1} - \ln c_t] &= -\sigma\Delta c_{t+1}^* - \Delta rer_{t+1} \\ \therefore \Delta c_t &= \Delta c_t^* + \frac{1}{\sigma} \Delta rer_t \end{aligned} \quad (8.5)$$

which can also be written in level form:

$$C_t = \mathcal{G}_{t-1} C_t^* RER_t^{1/\sigma} \quad (8.6)$$

# International Risk Sharing

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where  $g_{t-1} = C_{t-1} / (C_{t-1}^* RER_{t-1}^{1/\sigma})$  represents initial conditions.

Equation (8.5) advances two startling propositions:

- Consumption growth in any given country should be perfectly correlated with world consumption growth, once we adjust for fluctuations in the real exchange rate.
- Holding world consumption ( $C^*$ ) constant, consumption growth should rise with a real depreciation of the home currency, and more so the smaller risk aversion is.

# International Risk Sharing

- A lot of work has gone to test the growth correlations in (8.5) or the corresponding level relationship (8.6).
- A (somewhat outdated but still useful) summary of the evidence for advanced economies is in O-R, table 5: international correlations in consumption are non-trivial but correlations in income are still generally high.
- Essentially, what it says is that domestic consumption is still too sensitive to domestic income relative to what theory predicts.



# International Risk Sharing

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- This is odd since one of the supposed benefits of international financial integration is to increase risk sharing.
- And the evidence is that international financial integration greatly increased to all countries.
- This matters for risk sharing because greater financial linkages imply that a shock in country A affects the wealth of residents in country B, implying that  $\text{corr}(C_A, C_B)$  should rise.
- That is, greater internationalization of balance sheets helps hedge domestic risks! Domestic shocks become more like global shocks.

# International Risk Sharing

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- Yet, more recent evidence (Kose, Prasad and Terrones, 2009) indicates greater international risk sharing for advanced countries.
- In particular, these  $\text{cor}(\Delta c, \Delta c^*)$  have increased for those countries since the mid-1980s, what KPT call “globalization period”, as cross-border capital flows have become less restricted.
- The problem is more the much more limited degree of risk sharing across emerging markets: this is not only much lower than across advanced countries but also shows no sign of increasing on average.

# International Risk Sharing

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**Correlations between Domestic and World consumption Growth**  
(medians for each country group)

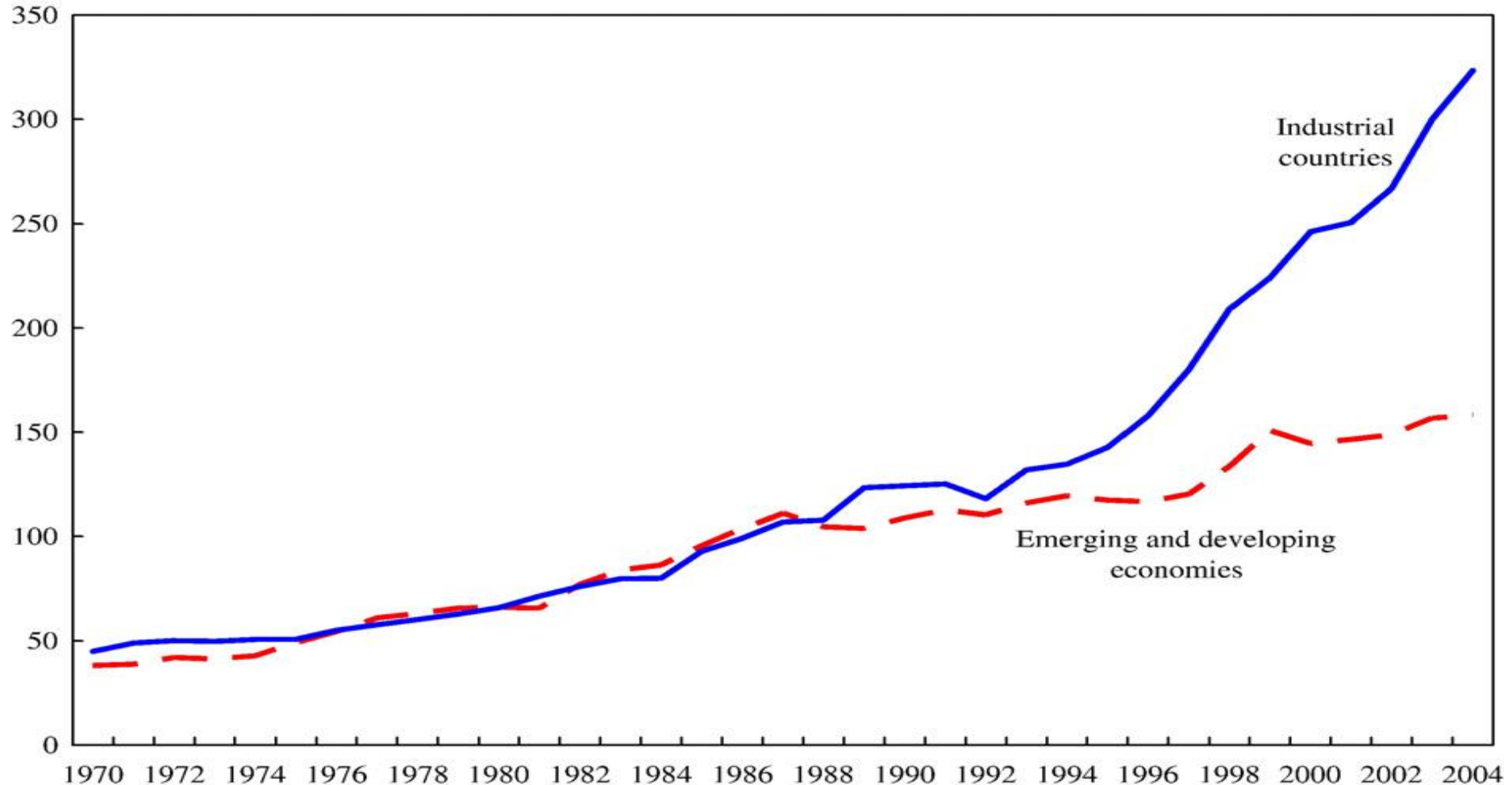
	<b>1961-2004</b>	<b>Bretton- Woods</b>	<b>Common Shocks</b>	<b>Globalization</b>
All Countries	<b>0.14</b> [0.04]***	<b>0.07</b> [0.05]	<b>0.2</b> [0.05]***	<b>0.07</b> [0.03]**
Industrial Countries	<b>0.5</b> [0.05]***	<b>0.22</b> [0.14]	<b>0.47</b> [0.11]***	<b>0.52</b> [0.10]***
Developing Countries	<b>0.03</b> [0.03]	<b>0.03</b> [0.05]	<b>0.04</b> [0.07]	<b>-0.03</b> [0.04]
Emerging Countries	<b>0.09</b> [0.04]*	<b>0.05</b> [0.09]	<b>0.02</b> [0.09]	<b>-0.11</b> [0.06]

From Kose, Prasad, and Terrones, 2009.

# International Risk Sharing

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## International Financial Integration [(A+L)/GDP]



From Lane and Milesi-Ferretti (2007)

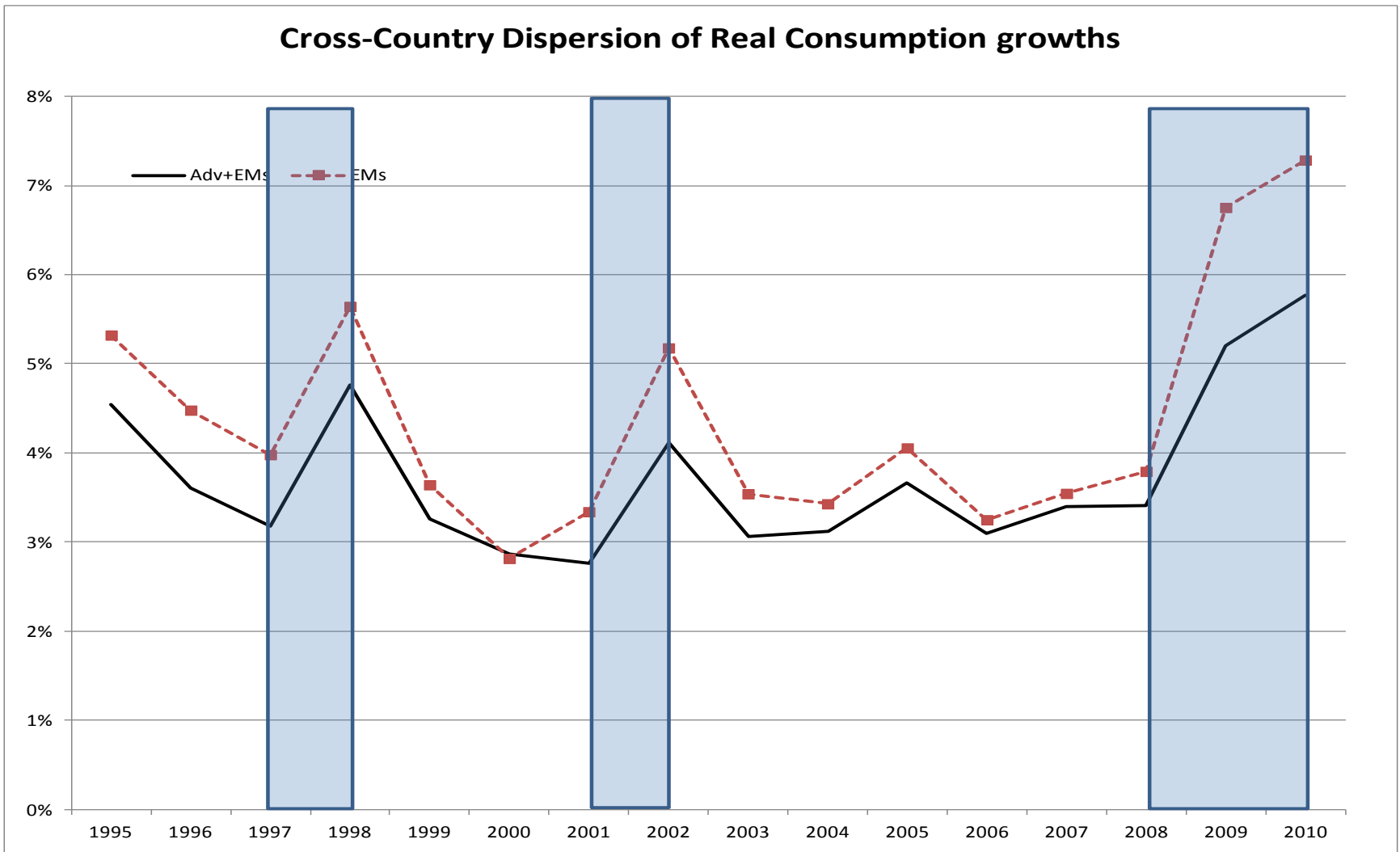
# International Risk Sharing

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- But in EMs it is clear that international financial integration still has quite some way to go.
- But the problem is also in the composition of external liabilities: a lot of it is still in debt rather than equity instruments.
- And is clear from the figure below that the cross-sectional dispersion in consumption growths take place around financial/debt crises.
- So, as noted in the KPT paper, it seems like that EMs have not yet benefitted more fully from the risk sharing benefits of financial globalization because of the high debt component limits it.

# International Risk Sharing

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# International Risk Sharing

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This takes us straight into issue of sovereign risk and external debt crises

To cement basic concepts let's first look at the simple two-period sovereign risk model. The references are O-R's (ch.6) and Catão and Kapur (IMF staff papers, 2005)

We then will move on the more elaborate infinite horizon model of Arellano (AER, 2008) who builds on the earlier seminal work of Eaton and Gersovitz (1981).

We will conclude with a discussion of the empirical determinants of external debt crises.

# Sovereign Risk: The Basic Model

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- Single good, two periods
- Sovereign country contracts  $P$  or borrows  $D$  in  $t=1$  and repays or defaults on this contract in  $t=2$ , when the world ends.
- To simplify, it cares only about period-2 utility:

$$U_1 = Eu(C_2) \quad (8.7)$$

- Output in  $t=2$  is stochastic and the country's total income will be output (GDP) plus any interest income from borrowing and saving the borrowing proceeds in  $t=1$ :



# Sovereign Risk: The Basic Model

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$$Y_2(D) = \bar{Y} + \varepsilon + RD \quad (8.8)$$

where  $\varepsilon$  has zero mean.

- In the case of an equity-type contract (as in O-R),  $D=0$  so

$$Y_2(D) = \bar{Y} + \varepsilon \quad (8.8a)$$

- Lenders/insurers operate in a competitive market and are risk neutral so:

$$\sum_{i=1}^N \pi(\varepsilon_i) P(\varepsilon_i) = 0 \quad (8.9)$$

# Sovereign Risk: The Basic Model

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Full Insurance Case: The country can commit and pay any  $P \leq Y_2$  as required by the equity type contract in  $t=1$ .

With  $P(\varepsilon) = \varepsilon$ :  $C_2(\varepsilon) = Y_2 - P(\varepsilon) = Y_2 - \varepsilon = \bar{Y}$

So, so consumption is fully smoothed.

But when  $\varepsilon > 0$ , the country has to make a payment to foreigners and can thus be tempted to renege on that.

In other words, the above contract needs to be made **incentive-compatible**.

# Sovereign Risk: The Basic Model

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In sovereign risk models, it is common to assume that there is a penalty for defaulting on a contract.

In finite horizon models (as well as some infinite horizon ones, as we shall see), the penalty is an output loss =  $\eta Y_2$ . Thus:

$$P(\varepsilon_i) \leq \eta(\bar{Y} + \varepsilon_i) \quad (8.10)$$

If so, the incentive compatible contract can be solved as follows:

# Sovereign Risk: The Basic Model

$$\max \sum_{i=1}^N \pi(\varepsilon_i) u[C_2(\varepsilon_i)] = \sum_{i=1}^N \pi(\varepsilon_i) u[\bar{Y} + \varepsilon_i - P(\varepsilon_i)] \quad (8.11)$$

st. (8.9) and (8.10). Then set-up the Lagrangian:

$$\begin{aligned} L = & \sum_{i=1}^N \pi(\varepsilon_i) u[\bar{Y} + \varepsilon_i - P(\varepsilon_i)] - \sum_{i=1}^N \lambda(\varepsilon_i) [P(\varepsilon_i) - \eta(\bar{Y} + \varepsilon_i)] \\ & + \mu \sum_{i=1}^N \pi(\varepsilon_i) P(\varepsilon_i) \end{aligned}$$

# Sovereign Risk: The Basic Model

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Differentiate wrt  $p$ :

$$\pi(\varepsilon)u'[C_2(\varepsilon)] + \lambda(\varepsilon) = \mu\pi(\varepsilon) \quad (8.12)$$

$$\lambda(\varepsilon)[\eta(\bar{Y} + \varepsilon) - P(\varepsilon)] = 0 \quad (8.13)$$

If  $\lambda(\varepsilon) = 0$ , the constraint is never binding, so the country can ensure smooth consumption. If not, a positive  $\lambda$  multiplier may imply uneven consumption across realizations of the output shock.

# Sovereign Risk: The Basic Model

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- Clearly, for low values of  $\varepsilon$ , repayment is not an issue so the constraint never binds and  $P(\varepsilon) = P_0 + \varepsilon$ , and  $u'(Y - P_0) = \mu$ .
- The critical step is to compute a threshold value  $\varepsilon = e$  above which the constraint starts binding. That is for  $\varepsilon$  above  $e$ ,  $\lambda(\varepsilon) > 0$ .

This definition of  $e$  implies:

$$\begin{aligned} Y - P_0 &= \bar{Y} + e - \eta(\bar{Y} + e) = (1 - \eta)(\bar{Y} + e) \\ \therefore P_0 &= \eta\bar{Y} - (1 - \eta)e \end{aligned} \tag{8.14}$$

# Sovereign Risk: The Basic Model

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We can now draw the repayment curve:

$$P(\varepsilon) = \begin{cases} \eta(\bar{Y} + e) + \varepsilon - e & \text{for } \varepsilon \leq e \\ \eta(\bar{Y} + \varepsilon) & \text{for } \varepsilon \geq e \end{cases} \quad (8.15)$$

Clearly, the repayment curve will be 45 degree sloped until  $e$  and then its slope =  $\eta$ .

Consumption will be flat until  $e$  and then will rise proportionally to  $(1-\eta) \varepsilon$ .

# Sovereign Risk: The Basic Model

What remains to be do is to pin-down  $e$ . This is done by assuming a distribution for  $\varepsilon$  and using the lender's break-even condition (8.9).

We are going to see how this is done shortly in the context of a debt (rather than equity contract) but see example in O-R 6.1.1.4 for how  $e$  is calculated.

A key point: Default in this model, with an equity-type of contract, takes place during “good times”, i.e.,  $\varepsilon > e$ . However, we shall we that this is not typically the case! In the model that follows, we shall see a different prediction.



# Sovereign Risk: The Basic Model

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- Now consider a model with a **debt contract**.
- Debt rather than equity type of contracts can arise for different reasons, costly monitoring of  $\varepsilon_i$  being a chief reason.  
 $\varepsilon$
- We stick to eqs. (8.7), (8.8) and the recovery technology in eq. (8.10), except for a change in the latter to take into account the size of the default.
- The model sketched is fully developed in Catão and Kapur (2005).

# Sovereign Risk: The Basic Model

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- Now the country borrows  $D$  and promises to repay  $R_L D$ .
- The commitment problem now arises over lower realizations of  $\varepsilon$ . That is when the country has a problem to come up with  $R_L D$  when the shock is negative.
- This can be optimal: **default can help smooth the  $Y$  shock as the country runs away with  $(1 - \eta)RD$** . If  $(1 - \eta)RD > \eta(\bar{Y} + \varepsilon)$ , there is some smoothing. Easily met if  $\varepsilon \ll 0$  and  $\eta < 1$ !
- So, with a debt contract, payment takes the form of:

$$P(\varepsilon, R_L, D) = \text{Min}[R_L D, \eta Y_2(D)]$$

# Sovereign Risk: The Basic Model

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So, we have:

$$P(\varepsilon, R_L, D) = \begin{cases} R_L D & \text{for } e \leq \varepsilon \leq \varepsilon_m \\ \eta[\bar{Y} + \varepsilon + RD] & \text{for } -\varepsilon_m \leq \varepsilon < e \end{cases} \quad (8.16)$$

where, as before  $e$  is the critical threshold between default and full repayment of contractual obligations:

$$e(R_L, D) \equiv \frac{[R_L - \eta R]D}{\eta} - \bar{Y} \quad (8.17)$$

$R$  being the risk-free interest rate.

# Sovereign Risk: The Basic Model

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- While the borrower loses a fraction  $\eta$  of its income upon defaulting, this doesn't mean that the lender will fully capture it.
- In earlier models (Cohen and Sachs, 1986), it was assumed that this was lost (the so-called deadweight losses of default).
- It is reasonable to assume that some of it is recovered by lenders (e.g., through gunboats or vulture funds)
- Here we assume a default of size  $S$  imposes a cost  $(1+q)S$  on the lender to recover the  $\eta$  income share.

# Sovereign Risk: The Basic Model

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- Hence, in case of default the net return to lenders will be:

$$P^*(\varepsilon, R_L, D) = R_L D - (1+q)S(\varepsilon, D). \quad (8.18)$$

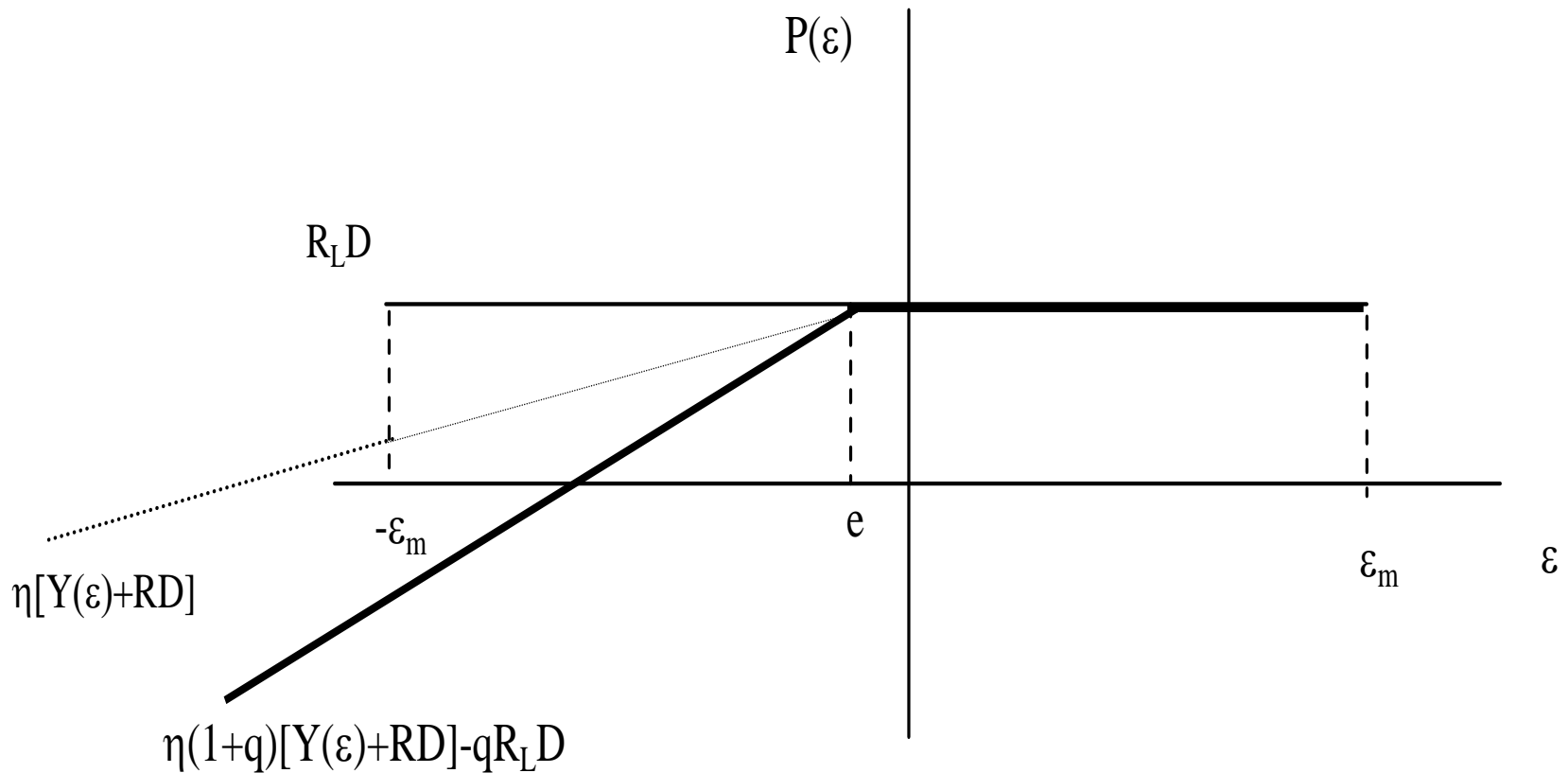
where  $q$  is a parameter that captures bargaining power between borrowers and lenders over the post-default income.

So, the payment schedule to lender will look like this:

# Sovereign Risk: The Basic Model

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Return to Lenders



# Sovereign Risk: The Basic Model

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For a continuous distribution, and sticking to the assumption that competitive lenders are risk neutral and break-even:

$$\int_{-\varepsilon_m}^{\varepsilon_m} P^*(\varepsilon, R_L, D) \pi(\varepsilon) d\varepsilon = RD \quad (8.19)$$

where  $(-\varepsilon_m, \varepsilon_m)$  is the support of the distribution.

Note that

$$\int_{-\varepsilon_m}^{\varepsilon_m} \pi(\varepsilon) d\varepsilon = 1 \quad (8.20)$$

# Sovereign Risk: The Basic Model

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For a continuous distribution, and sticking to the assumption that competitive lenders are risk neutral and break-even:

$$\int_e^{\varepsilon_m} R_L D - \int_{-\varepsilon_m}^e [\eta(1+q)(Y(\varepsilon, D) + RD) - qR_L D] \pi(\varepsilon) d\varepsilon = RD \quad (8.21)$$

where  $(-\varepsilon_m, \varepsilon_m)$  is the support of the distribution.

Using (8.20) in the above yields:

$$(R_L - R)D = \int_{-\varepsilon_m}^{e(R_L, D)} \eta(1+q)[e(R_L, D) - \varepsilon] \pi(\varepsilon) d\varepsilon \quad (8.22)$$



# Sovereign Risk: The Basic Model

## Proposition 1

- (a)  $R_L(D)$  is well-defined for levels of debt in some bounded interval  $[0, D_{\max})$ , where  $D_{\max}$  depends, inter alia, on the probability distribution of shocks,  $\pi(\varepsilon)$ .
- (b)  $R_L(D) = R$  for  $D \in [0, \frac{\eta}{1-\eta} \frac{\bar{Y} - \varepsilon_m}{R}]$ . For higher values of  $D$ ,  $R_L(D)$  exceeds  $R$  and is strictly increasing in  $D$ .
- (c)  $R_L(D)$  is increasing in the variance of shocks.

# Sovereign Risk: The Basic Model

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- $D_{max}$ : Given this, the gain from defaulting =  $RL D - \eta Y + \varepsilon + RD$  is increasing in  $D$ .
- *If so, then for some  $D$  high enough, the borrower will default with probability one. This implies that, given  $Y$  and  $R$ , there exists some  $D$ , call it  $\max D$ , such that the lender cannot break-even for debt levels beyond this.*
- To prove proposition 1.b, consider the worse possible shock  $\varepsilon = -\varepsilon_m$ . If it does not default then, it will never default, and with  $RL=R$  in (8.17) and  $e = -\varepsilon_m$  this yields the upper limit of  $D$  under no default.

# Sovereign Risk: The Basic Model

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- To prove (1.b), total differentiate (8.21):

$$dR_L \left[ -\int_{-\varepsilon_m}^e qD\pi(\varepsilon)d\varepsilon + D\int_e^{\varepsilon_m} \pi(\varepsilon)d\varepsilon \right] +$$
$$dD \left[ n(1+q)R\int_{-\varepsilon_m}^e \pi(\varepsilon)d\varepsilon - qR_L\int_{-\varepsilon_m}^e \pi(\varepsilon)d\varepsilon + \int_e^{\varepsilon_m} R_L\pi(\varepsilon)d\varepsilon - R \right] = 0$$

- and use: 
$$\phi(R_L, D) = \int_{-\varepsilon_m}^{e(R_L, D)} \pi(\varepsilon)d\varepsilon$$

$$\therefore 1 - \phi = \int_{e(R_L, D)}^{\varepsilon_m} \pi(\varepsilon)d\varepsilon$$

- To obtain after some simple manipulation:

$$\frac{dR_L}{dD} = \frac{1}{D} \left( -R_L + R + R(1-\eta) \frac{(1+q)\phi}{1-(1+q)\phi} \right) \quad (8.23)$$

# Sovereign Risk: The Basic Model

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- Using the break-even condition, it can then be shown that the term in parenthesis is positive. Clue: start by showing that:

$$RD > \phi(\eta(1+q)RD - qR_L D) + R_L D(1-\phi)$$

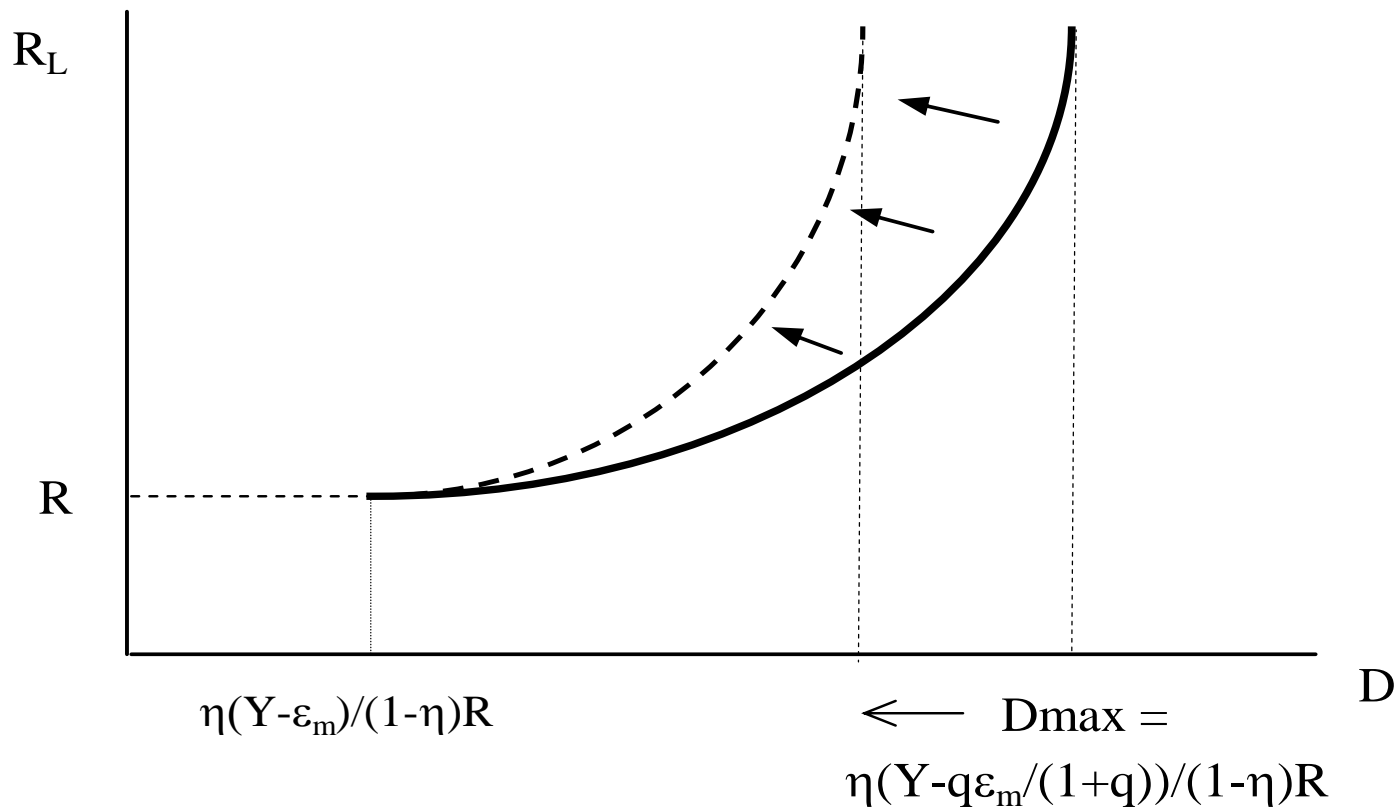
and then show that the numerator of (8.23) is positive

- On 1c, the expected repayment on the left hand side of (8.22) is concave in shocks. By Jensen's inequality, an increase in the volatility of shocks lowers the expected value of repayments for a given. To restore the break-even requirement,  $R_L$  must rise.

# Sovereign Risk: The Basic Model

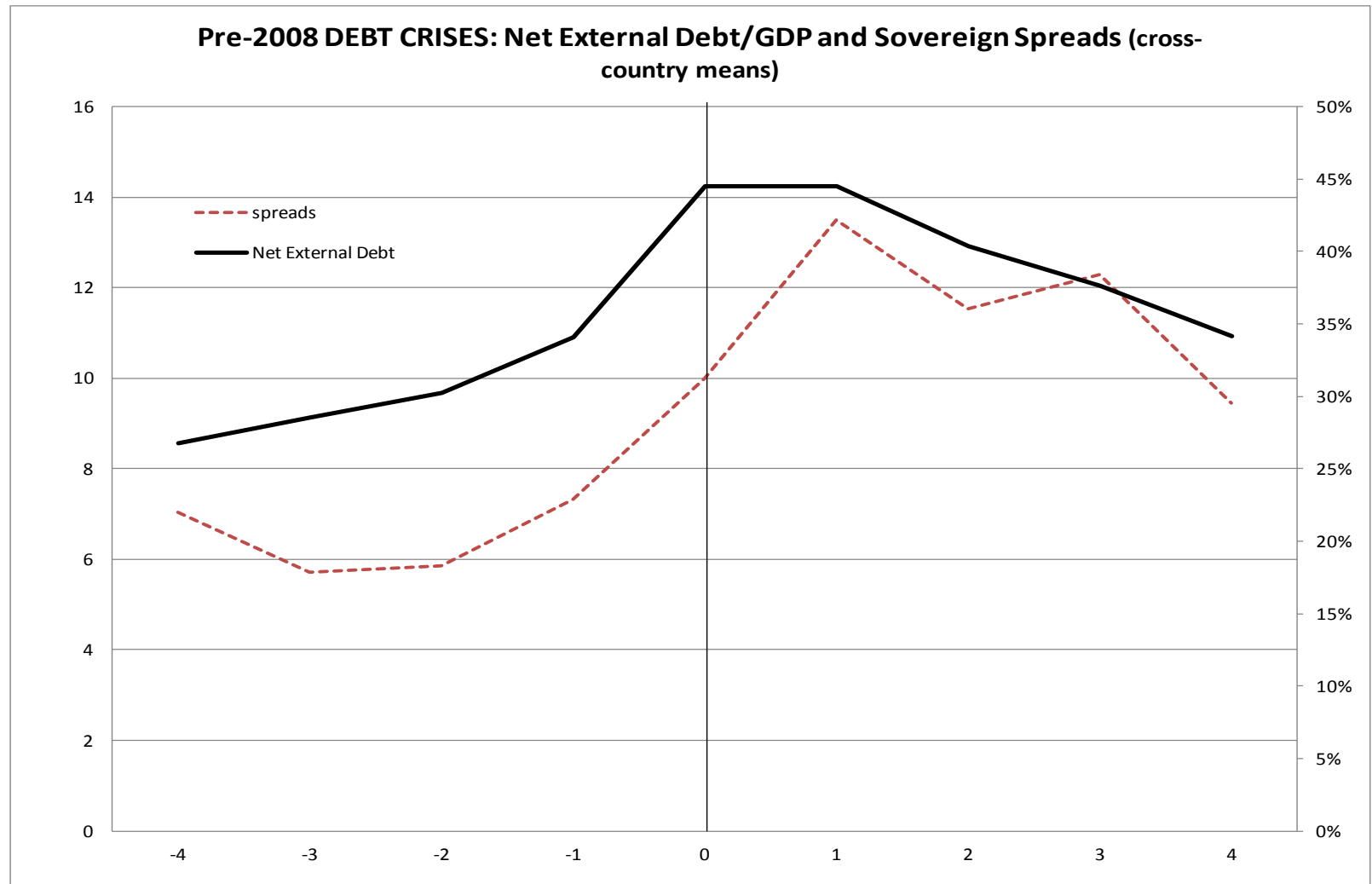
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## Effects of Volatility on Spreads and Borrowing Ceilings



# Sovereign Defaults: Stylized Facts 1

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# Sovereign Defaults: Stylized Facts 2

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# Infinite Horizon Set-up

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- The two period debt model just studied requires a punishment or debt recovery technology for default not to become a trivial decision.
- As we move to a multi-period set-up, there is another punishment for default which can motivate sovereign lending.
- Such a punishment is exclusion from world capital markets. The longer the exclusion, the longer the country cannot smooth consumption via international borrowing.
- If domestic income volatility is high, then this cost can be non-trivial.



# Infinite Horizon Set-up

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- In the model we now study (Arellano, AER 2008), the two punishments are present.
- This helps the model to generate higher debt levels that are closer to real world counterparts.
- While more evolved, the Arellano model shares with the previous one three main results:
  - i. Default takes place in “bad times”
  - ii. The sovereign spread (RL-R) rise on debt.
  - iii. The spread rises on income volatility

# Infinite Horizon Set-up

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## Arellano (2008)

- Small Open Economy where output follows a Markov chain with transition function  $f(y, y')$
- Households:  $\max. E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$  (8.24)
- Sovereign holds internationally traded asset  $B'$ , which here takes the form of a **one-period discount bond** with price  $q$ .
- The government can default on its obligations ( $-B'$ ). Lenders cannot, i.e., there is a **one-sided commitment problem**.

# Infinite Horizon Set-up

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- The bond price  $q(B', y)$  is endogenous, reflecting default risk.

If  $B' \geq 0$ , the government is a net creditor, saving  $q(B', y)B'$  at  $t$  and receiving  $B'$  at  $t+1$ . As we shall, default risk is then zero.

If  $B' < 0$ , the government is a net debtor, borrowing  $q(B', y)B'$  at  $t$  and repaying  $B'$  at  $t+1$ . So, obligation value  $B'$  is state-invariant; only default makes payment state dependent.

- As usual in these models, the government is benevolent, transferring lump-sum proceeds of bond trading to domestic agents.

# Infinite Horizon Set-up

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If **no** default, the country's budget constraint is:

$$c + q(B', y)B' = y + B$$

If it defaults, then the country enter autarky and:

$$c = h(y) \leq y$$

The idea is that default entails income losses, so the country's national income falls relative to its previous (no-default) output generation process.

As in Eaton and Gersovitz (1981), defaults entails market exclusion. But unlike them, Arellano assumes it as temporary.

# Infinite Horizon Set-up

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As in Aguiar and Gopinath (JIE, 2006), for each  $t$  after default, there is an **exogenous** probability  $\theta$  of market re-entry .

## Lenders:

- As in the two period model, they operate in competitive market and are risk neutral, so they seek to break-even period by period.
- Have access to a risk free asset that pays  $R=1+r$
- Full information: lenders observe  $y$  and  $f(y',y)$

# Infinite Horizon Set-up

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- Expected profits by lenders are:

$$qB' - \frac{1 - \delta}{1 + r} B'$$

Note that now  $\delta$  is the probability of default (what before we called  $\varphi$ )

- If  $B' \geq 0$ , then  $\delta = 0$ , so  $q = 1/R$ .
- If  $B' < 0$ , then  $\delta = [0, 1]$ ,  $q = 1/R_L = (1 - \delta)/(1 + r)$ .

Given  $B'$  and  $y'$ , the bond price satisfies:  $q(B', y) = \frac{1 - \delta(B', y)}{1 + r}$

## Infinite Horizon Set-up

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- We can now define the so-called sovereign spread  $r_L - r$ :

$$\frac{1 + r_L}{1 + r} = \frac{1}{1 - \delta}$$

$$\therefore r_L - r = \delta(1 + r_L) = \frac{\delta}{q}$$

$$\therefore r_L - r = \frac{\delta R}{1 - \delta}$$

So, as intuitive, the sovereign spread is inversely related to bond price  $q$ , and is proportional to the probability of default  $\delta$  and increases on the risk-free interest rate  $r$ .

# Infinite Horizon Set-up

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- As in the two-period set-up, default is simply a cost vs. benefit decision, given the objective function of max (8.24).
- The aim here is to show that it can take place in equilibrium.
- To this aim, define the following value function:

$$V^o = (B, y) = \max \{V^c(B, y), V^d(y)\}$$

Where  $V^c$  is the continuity value (no-default) and  $V^d$  is the value of default.



# Infinite Horizon Set-up

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- Now define:

$$V^d = u(y^{def}) + \beta \int_{y'} [\theta V^0(0, y') + (1-\theta)V^d(y')] f(y', y) dy'$$

Where  $1-\theta$  is the probability of staying in default.

$$V^c = \max_{B'} \left\{ u(y - q(B', y)B' + B) + \beta \int_{y'} V^0(B', y') f(y', y) dy' \right\}$$

Note that the continuity value incorporate the possibility of that the government may find optimal to default in the future.

Also note that the no-Ponzi condition is implicit in this policy function by assuming the gov. faces a lower bound  $B' \geq -Z$ .

# Infinite Horizon Set-up

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- The government policy has repayment  $[A(B)]$  and default  $[D(B)]$  sets which will depend on  $y$ 's and  $B$ :

$$A(B) = \{y \in Y : V^c(B, y) \geq V^d(y)\}$$

$$D(B) = \{y \in Y : V^c(B, y) < V^d(y)\}$$

Letting  $s = \{B, y\}$  be the aggregate states, we can now define the recursive equilibrium of this model economy.

In this equilibrium the government, foreign creditors, and households act sequentially.

# Infinite Horizon Set-up

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- The government starts off with assets  $B$  and observes income  $y$ .
- It then decides whether to pay or not the debt by evaluating the respective value functions.
- If it repays, it takes  $q(B,y)$  and chooses  $B'$ .
- If it defaults, it goes into (temporary autarky)
- The representative agent then consumes.

# Infinite Horizon Set-up

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## Equilibrium Definition:

The recursive equilibrium for this economy is defined as a set of policy functions for  $c(s)$ , government asset holdings  $B'$ , repayment sets  $A(B)$  and default sets  $D(B)$ , and bond price function  $q(B',y)$  that are consistent with the government's optimization, creditor's expected zero profit condition, and the household resource constraint.

# Infinite Horizon Set-up

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- Similarly as before, default probabilities  $\delta$ 's and default sets  $D(B')$  are related as follows:

$$\delta(B', y) = \int_{D(B')} f(y', y) dy'$$

- As in the two period model, default sets are rising on negative assets (more debt) [Arellano's proposition 1]
- She then considers the particular case where endowment shocks are i.i.d  $\rightarrow$  this makes  $q(B')$  independent of the shock realization as  $t$  shocks give no information future output and hence default probabilities. Also assume  $h(y)=y$ .

## Infinite Horizon Set-up

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- Proposition 2: *If for some  $B$  the default set is non-empty  $D \neq \emptyset$ , then there are no contracts available such that the economy experiences capital inflows  $B - q(B')B' > 0$ .*

As homework, prove this by contradiction.

Intuition: if  $D(B)$  is non-empty and there were to exist a contract that allowed the government to roll over debt, then it is always optimal for the sovereign to take more debt and default tomorrow on a higher debt stock. This cannot be the case.

# Infinite Horizon Set-up

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- Proposition 3: *Default incentives are stronger the lower the endowment.*

As in the two-period model, this result comes from utility concavity in consumption and that under no default the economy experiences a capital outflow: repayment is more costly when  $Y$  is low.

As discussed earlier, this contrasts with the participation constraint model (equity-type contract) wherein the incentives to default are ***higher*** during good times.

# Infinite Horizon Set-up

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- As in the two-period model, we can devise a repayment function dividing the  $(yB)$  space into repayment and default regions.
- Default sets are then characterized a close interval where the upper bond is a function of assets  $[y_{\min}, y^*(B)]$ .
- Each contract  $\{q(B')B'\}$  changes consumption today by the product  $q(B')B'$ .
- With iid shocks, the set of contract are the same for every  $t$ , and characterized by (where  $F^*$  the cummulative):

$$q(B')B' = \frac{1}{1+r} [1 - F(y^*(B'))] B'$$



# Infinite Horizon Set-up

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- Arellano then calibrates the model using CARA utility, a sensible (but yet ad hoc output cost function  $h(y)$ ) and business cycle “stylized facts” for Argentina.
- This includes highly persistent (rather than iid)  $y$  shocks.
- Consistent with data, she finds that spreads are counter-cyclical and  $B^*(y_{\text{high}}) < B^*(y_{\text{low}})$ .

## Infinite Horizon Set-up

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- Note that to help match model and data regarding debt levels in equilibrium  $\beta < 1/R$ , making even cheaper to borrow in good times.
- This also helps ensure that the economy is borrowing constrained in bad times, so matching the real world fact that there is capital outflow when output is lower.
- That is, capital outflows ( $y - c > 0$ ) can occur in recessions despite that is when borrowing is mostly valued – we saw those two forces operating also in the 2-period set up!

## Infinite Horizon Set-up

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- Yet, not all is blue sky as these models have a hard time to generate realistic mean debt levels, except for some twicking in the parameterizations: low  $\beta$ ,  $\theta$ , and low output under autarky.
- The model also generate mean spreads that are typically lower than in real world data.
- This is a more general problem of risk neutral pricing in which the relationship between default probability (3% in her calibration) and mean spreads are “hard-wired” in the calibrations.

## Infinite Horizon Set-up

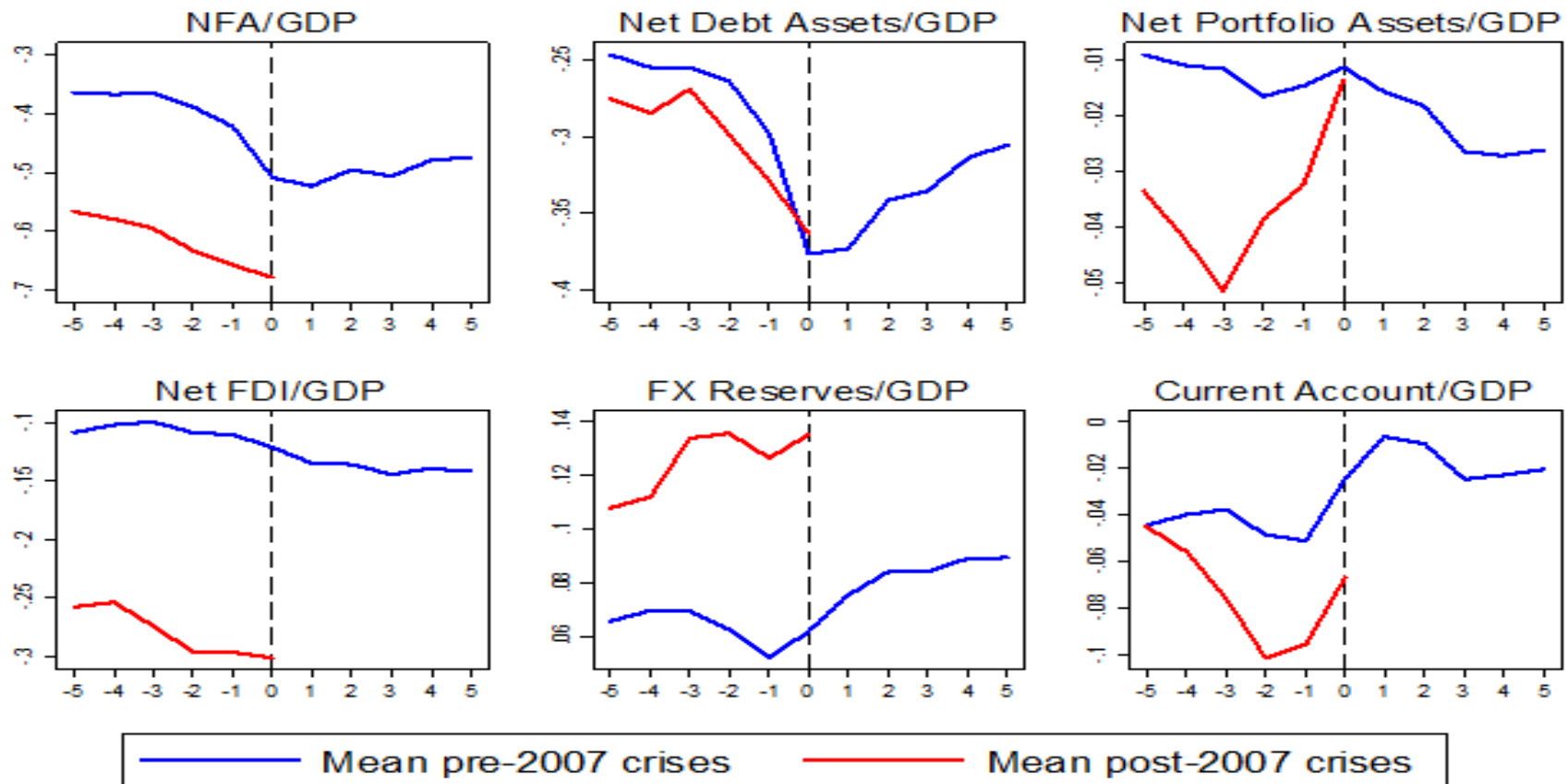
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- To sum up, the sovereign risk models studied help conceptualize some main issues regarding the decision to borrow and default.
- They also successfully rationalize the relationship between  $B$  and  $r_L - r$ ,  $\text{cov}(r_L - r, y) < 0$ , and the fact that defaults typically occur in bad times.
- Yet, they have clear limitations too. E.g., the decision to borrow is multifaceted (smoothing consumption is not the only one and perhaps not even the main one), information is not fully symmetric, etc.
- In particular, such models cannot rationalize long bouts of spread convergence and divergence across countries with disparate  $B$  and  $\Delta Y$  – e.g. the eurozone during the spread convergence of 1999-2007 and the decoupling of 2008-2012 being an illustration (cf. slide 47).

# What Do We Know about Drivers of External Crises?

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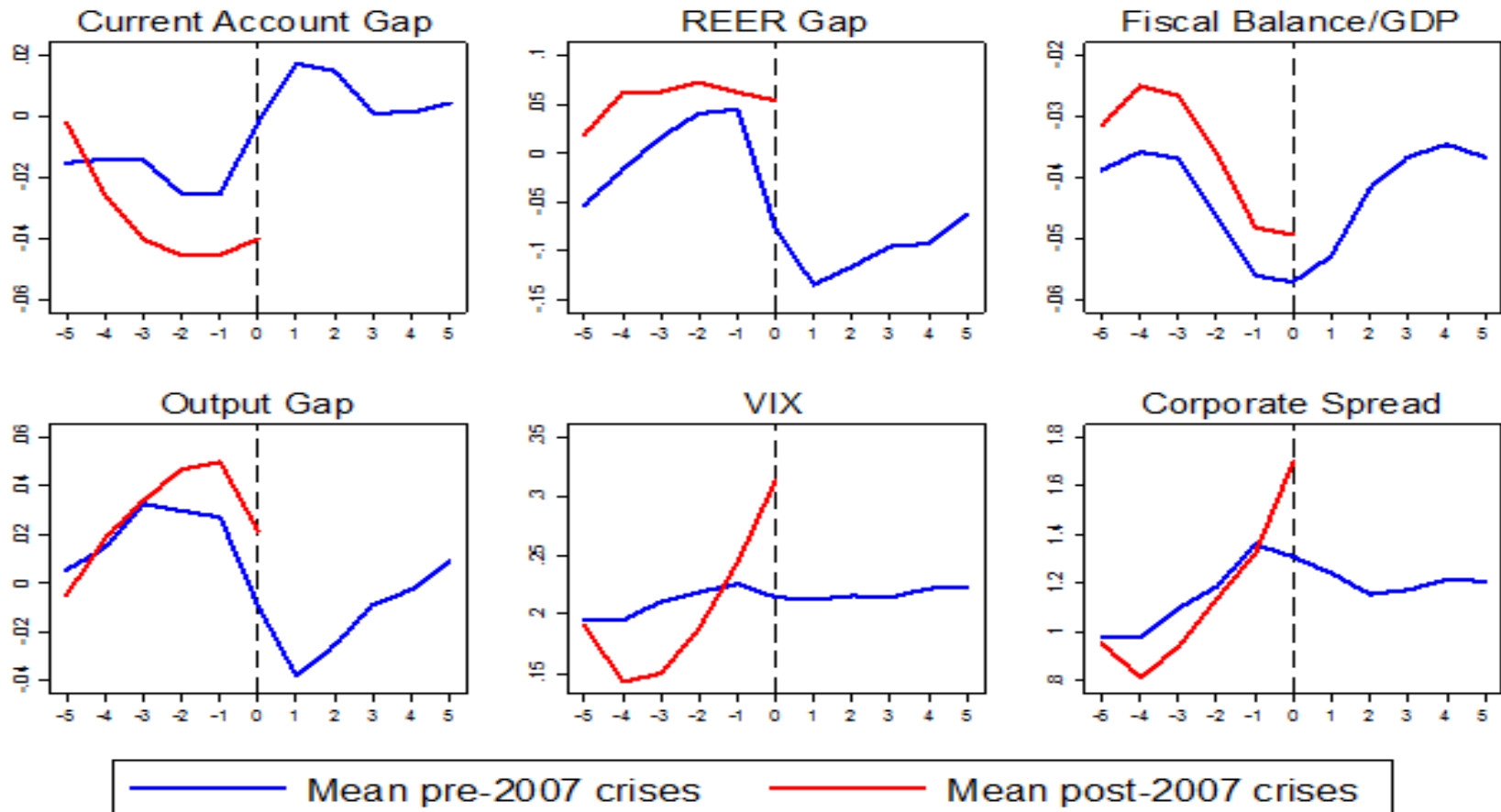
## Net Foreign Assets (NFA) and Components Around External Crises



Source: Catão and Milesi-Ferretti (2013)

# What Do We Know about Drivers of External Crises?

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Source: Catão and Milesi-Ferretti (2013)

# What Do We Know about Drivers of External Crises?

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- Clearly, default events and external crises more broadly (which in C-MF includes averted defaults by multilateral bail-outs) are typically associated with NFA going into negative territory (higher net foreign liabilities)
- Much of it being net debt liabilities.
- The speed of NFA changes (current account balance) clearly matters – crises have been typically associated with CA deficits of more than 4% of GDP.
- Reserves fall and crises typically take place with reserves still being positive (as in the Krugman “first-generation” model) .

# What Do We Know about Drivers of External Crises?

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- Also as in “first generation” crisis models – crises have been typically associated with further deterioration of the *overall* fiscal deficit...
- And tend to take place under rising global stock market volatility and distressed global financial market conditions more generally (as proxied by the VIX index and the US corporate spread).
- Overall, we can show that a probit model containing these fundamentals can do a good job in out-of-sample crisis prediction.
- Summing up: what we have learned about macro fundamentals appears to matter a great deal!



# What Do We Know about Drivers of External Crises?

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Probit Model with Macro Fundamentals:  
Out-of-sample Predictions of External Crisis Probabilities for post-2007 period

