Lecture VII:

Monetary Policy and the Exchange Rate

Uncovered and Covered Interest Rate Parity

As with PPP, a key arbitrage conditions in international macroeconomics is the *uncovered interest rate parity* ***(UIP)*** condition:

$$(1+i_t) = (1+i_t^*)E_t\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right) \qquad (7.1)$$

where as before ε_t is spot exchange rate.

Think of it as follows. The home country has an interest rate of e.g. i=9% a year in reais, whereas the foreign country has i*=1% a year in US$. So, if the exchange rate is expected to stay constant, it becomes profitable to borrow in US$ at 1% and lend at home at 9%, yielding an arbitrage gain of 8%.

Uncovered and Covered Interest Rate Parity

But this cannot be an "equilibrium" condition when international capital markets are free of restrictions, in the same way that the same good cannot have perpetually a different price from the same good abroad when goods can move freely across borders.

So, either the i-i* will adjust or the exchange rate will depreciate. E.g. the exchange rate first appreciates as dollars flow in people convert dollars into reais to buy the domestic bond and then depreciates when people pay back their dollar debts by selling the reais accruing at the maturity of the domestic bond.

Uncovered and Covered Interest Rate Parity

It is common to write (7.1) in linear form applying the log transformation and using the approximation ln(1+x)~x:

$$i_t \simeq i_t^* + [E_t(e_{t+1}) - e_t] = i_t^* + E_t \Delta e_{t+1} \qquad (7.1)$$

which clearly indicates that in countries where the nominal interest rate is higher, the currency is expect to eventually *depreciate*.

In practice, however, this relationship does not hold too well in the data (see, e.g., Frankel and Rose, 1995)

Uncovered and Covered Interest Rate Parity

The reasons can be various, pertaining to the way expectations are formed, the existence of a risk premium associated with nominal exchange rate volatility, default risk, and capital controls.

So, in more general terms (7.1) is written as:

$$i_t \simeq i_t^* + E_t \Delta e_{t+1} + \zeta_t \qquad (7.2)$$

where ζ_t is meant to capture a risk premium which can be positive or negative, and possibly time-varying.

Uncovered and Covered Interest Rate Parity

In practice, it is possible for the investor to try to eliminate exchange rate uncertainty in international bond trade.

This is done by a forward contract where an investor promises to sell you dollars at a fixed parity to reais in t+1.

Abstracting from default risk and capital controls, this gives rise to the so-called *covered* interest rate parity (CIP):

$$1 + i_t \simeq (1 + i_t^*) \frac{F_t}{\varepsilon_t} \qquad (7.3)$$

Uncovered and Covered Interest Rate Parity

where F is the forward exchange rate at time t for the purchase of reais per unit of dollars at t+n, where n is the length of the maturing bond contract.

In equilibrium, with risk neutral investors and no capital controls, one should thus expect that:

$$F_t = E_t(\varepsilon_{t+1}) \qquad (7.4)$$

A key question is, again, whether that holds in practice!

Uncovered and Covered Interest Rate Parity

As with the pure UIP specification of equation (7.1), the results of empirical tests are not very favorable.

As a test, researchers have run the following regression, where f=ln(F):

$$e_{t+1} - e_t = a_o + a_1 \underbrace{(f_t - e_t)}_{\text{Forward premium}} + \upsilon_t \qquad (7.5)$$

υ_t: $i.i.d$ error

If forward markets are fully efficient, we should have:

$$a_o = 0 \qquad a_1 = 1$$

Uncovered and Covered Interest Rate Parity

Tests typically reject a_1=1. In fact, often the estimate is negative and sometimes statistically insignificant.

This implies that the forward rate is typically a biased predictor of future spot exchange rates.

One rationale is excessive exchange rate volatility.

Indeed, if future spot exchange rate e_{t+1} is log-normally distributed, we have:

Uncovered and Covered Interest Rate Parity

$$f_t = E_t(e_{t+1}) + \frac{1}{2}\mathrm{var}_t(e_{t+1}) \qquad (7.6)$$

So, the last term in (7.6) would be missing in tested equation (7.5).

Importantly, also note that we let variance term to be time-varying.

Equation (7.6) provides the basis for a rationale due to Eugene Fama (1984) for the “failure” of equation (7.5).

Uncovered and Covered Interest Rate Parity

Let's now look at the Fama rationale. In doing that, we follow the exposition in O-R, section 8.7.5.3.

First start by considering the OLS estimator of the slope coefficient a_1 in (7.5). Asymptotically we have:

$$p\lim(a_1) = \frac{\operatorname{cov}(f_t - e_t, e_{t+1} - e_t)}{\operatorname{var}(f_t - e_t)} \qquad (7.7)$$

Now define the risk premium (not to be confused with the forward premium f_t-e_t):

$$rp_t = f_t - E_t(e_{t+1}) \qquad (7.8)$$

Uncovered and Covered Interest Rate Parity

Add e to both sides so that:

$$f_t - e_t = E_t(e_{t+1}) - e_t + rp_t \qquad (7.9)$$

If markets are fully efficient, the following rational expectations restriction holds:

$$\text{cov}_t[e_{t+1} - E_t(e_{t+1}), \Omega_t] = 0 \qquad (7.10)$$

where Ω_t is the information set available to investors at t and includes the forward premium $f_t - e_t$, i.e:

$$E_t[(e_{t+1} - E_t(e_{t+1})(f_t - e_t)] = 0 \qquad (7.11)$$

Uncovered and Covered Interest Rate Parity

So, under rational expectations we can re-write (7.7) as:

$$p\lim(a_1) = \frac{\text{cov}[f_t - e_t, E_t(e_{t+1}) - e_t)]}{\text{var}(f_t - e_t)} \qquad (7.12)$$

The two key results due to Fama can now be readily shown:

- <u>Condition for $\hat{a}_1 < 0$</u>:

$$\text{cov}[f_t - e_t, E_t(e_{t+1}) - e_t] = \text{cov}[E_t(e_{t+1}) - e_t + rp_t, E_t(e_{t+1}) - e_t] < 0$$

$$\therefore \text{var}[E_t(e_{t+1}) - e_t] + \text{cov}[E_t(e_{t+1}) - e_t, rp_t] < 0$$

Uncovered and Covered Interest Rate Parity

Since variances are always positive, you can only get a negative slope coefficient a_1 if, and only if:

$$\operatorname{cov}[E_t(e_{t+1}) - e_t, rp_t] < 0 \qquad (7.13)$$

That is, the more the nominal exchange rate is "mis-aligned" today relative to what is expected to be tomorrow (given all relevant information available at t), the smaller the risk premium: i.e. forward rates will adjust to be closer to expected spot exchange rate.

Uncovered and Covered Interest Rate Parity

- Condition for $\hat{a}_1 < 1/2$:

Multiply both sides of (7.12) by ½ and use again (7.9) to obtain:

$$\mathrm{var}(rp_t) > \mathrm{var}[E_t(e_{t+1}) - e_t] \qquad (7.14)$$

This means that the volatility of the risk premium should be greater than the volatility of expected changes in the exchange rate.

This is a disturbing result as it highlights the difficulty of modeling exchange rate risk by the fundamentals governing $E(e_{t+1})-e_t$.

Uncovered and Covered Interest Rate Parity

Yet, if the nominal exchange rate follows a random walk so that $E_t(e_{t+1}) - e_t \sim 0$ then the risk premium will also be small.

Conversely, if the exchange risk premium is large, the puzzle is then why the expected changes in the exchange rate are so small, as implied by the random walk hypothesis.

Clearly, here there are significant differences between advanced and emerging markets regarding nominal exchange rate variability.

We will return to the issue of excessive E volatility shortly.

Basic Monetary Model of the Exchange Rate

- Armed with PPP and UIP, we can now readily develop a basic (but traditionally widely used) model of the *nominal* exchange rate.

- The first building block is a standard money demand function that we have seen in the first part of the course (the money demand function in the shopping time model of Ljungqvist & Sargent (2004) and as well as in the Cagan model:

$$m_t - p_t = -\eta i_t + \phi y_t \qquad (7.15)$$

Basic Monetary Model of the Exchange Rate

Now substitute the log-linear PPP an UIP equations of (6.2) and (7.1) into (7.15) to substitute out i and p and obtain:

$$(m_t - \phi y_t + \eta i_t^* + p_t^*) - e_t = -\eta(E_t e_{t+1} - e_t) \qquad (7.15)$$

$$\therefore \eta \underline{E_t e_{t+1}} - (1+\eta)\underline{e_t} + (m_t - \phi y_t + \eta i_t^* - p_t^*) = 0$$

This is an stochastic difference equation in the (log of) nominal exchange rate, where m, y, i* and p* are the exogenous, forcing variables. These are the so-called "fundamentals".

Basic Monetary Model of the Exchange Rate

To solve it, first ignore the stochastic part, assuming perfect forecast so that $E(e_{t+1})=e_{t+1}$.

To simplify the algebra, call $f = m_t - \phi y_t + \eta i_t^* + p_t^*$.

Thus we have:

$$e_t = \frac{f_t}{1+\eta} + \frac{\eta}{1+\eta} e_{t+1}$$

Iterating forward yields:

Basic Monetary Model of the Exchange Rate

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} E_t(m_t - \phi y_t + \eta i_t^* - p_t^*) + \lim_{T\to\infty} \left(\frac{\eta}{1+\eta} \right)^{T} e_{t+T}$$

As usual, we rule out the presence of speculative bubbles (the equivalent of Ponzi games), by setting the last term to zero, so the nominal exchange rate is given by:

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} E_t(m_t - \phi y_t + \eta i_t^* - p_t^*) \qquad (7.16)$$

Basic Monetary Model of the Exchange Rate

Some important "take-home" points from this equation:

- The nominal exchange rate today reflects the future evolution of its "fundamentals" (in this case money supply, output, the foreign interest rate and foreign price level).
- That is, **the exchange rate is essentially a forward-looking variable**.
- As such, conditional on the model, e today should help predict f!
- Eq. (7.16) also tells us what to expect on the direction of the responses of e to changes in the various fundamentals.

Basic Monetary Model of the Exchange Rate

- A loosening of monetary policy, i.e., higher m in the future implies that the exchange rate should depreciate (i.e., e rises).

- Conversely, a productivity improvement that raises y will tend to appreciate the nominal exchange rate (i.e. e falls).

- Consider now a rise in the foreign interest rate (i*) due to say the end of QE policies in the US. Assuming that p* remains about stable, this implies a increase in US ***real*** interest rate.

- The model says that tends to depreciate the home exchange rate.

Basic Monetary Model of the Exchange Rate

Since we are particularly interested here in the effect of changes in monetary policy on the exchange rate, let's examine on what the model says on sensitivity of e to changes in money supply.

As usual in solving the models, we make progress by assuming an exogenous stochastic process for the respective "state" variable. As in O-R (section 8.2.7), assume:

$$m_t - m_{t-1} = \rho(m_{t-1} - m_{t-2}) + \upsilon_t \qquad (7.17)$$

Basic Monetary Model of the Exchange Rate

As in O-R, assume to simplify that $-\phi y_t + \eta i_t^* - p_t^* = 0$ so we plug (7.17) into (7.16) and take expected differences to obtain:

$$E_t e_{t+1} - e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} E_t (E_t m_{t+1} - m_t)$$

$$= \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta\rho}{1+\eta} \right)^{s-t} \rho (m_s - m_{s-1})$$

$$= \frac{1}{1+\eta} \frac{(m_t - m_{t-1})}{1 - \frac{\eta p}{1+\eta}} = \frac{1}{1+\eta} (1+\eta) \rho \frac{(m_t - m_{t-1})}{1+\eta - \eta p}$$

Basic Monetary Model of the Exchange Rate

We can then invoke (7.15) to yield:

$$\frac{e_t - m_t}{\eta} = E_t e_{t+1} - e_t \qquad (7.18)$$

And then substitute out E(e$_{t+1}$)-e$_t$:

$$\frac{e_t - m_t}{\eta} = \frac{\rho(m_t - m_{t-1})}{1 + \eta - \eta p}$$

$$e_t = m_t + \frac{\rho\eta}{1 + \eta(1 - \rho)}(m_t - m_{t-1}) \qquad (7.19)$$

Basic Monetary Model of the Exchange Rate

Equation (7.19) states that the impact of monetary shocks (v) on the exchange rate will rise on

- The persistence of monetary shocks (higher ρ)

- On the semi-elasticity of money demand (η).

Since the last term in (7.18) is positive, this means that shocks to money growth have a more than proportional effect on the nominal exchange rate.

Testing the Nominal Exchange Rate Model

- Influential paper by Meese and Rogoff (1983) tests the flex- and sticky price monetary model based on out-of-sample performance.

Flex-price model: $e_t = m_t - m_t^* - \gamma(y_t - y_t^*) + \lambda(i_t - i_t^*)$

Sticky-price model: $e_t = m_t - m_t^* - \gamma(y_t - y_t^*) + \lambda(i_t - i_t^*) + \theta(E_t e_{t+1} - e_t)$

- Meese-Rogoff (1983) estimate these models for the DM-US$ and Yen-US$ over Mar73-Dec76 and compute the out of sample $\hat{e}_t$ for 1-, 3-, 6-, 12-months ahead

Testing the Nominal Exchange Rate Model

- They do the same for the random walk model $\hat{e}_{t+1} = e_t$
- They then compute the mean-squared error $\frac{1}{k}\sum_{j=1}^{k}(e_j - \hat{e}_j)^2$
- They then find that those monetary models cannot beat the random walk
- Others (Mark, 1995; Mark & Sul, 2001) have found, however, that a longer horizons and over a longer sample (in Mark 1973:II to 1991:IV), the flex-price monetary model tends to beat the random walk.

Testing the Nominal Exchange Rate Model

- Subsequent research indicates that both results are quite sample dependent.
- In general, it appears that the monetary model has an (small) edge out of sample, but only longer periods.
- Yet cumulatively this gain can be non-trivial; and non-linearities appear to be important.
- Others (Engel and West, 2006) question the meaningfulness of out-of-sample tests in the style of Meese and Rogoff.
- In short, while the jury is out, the flex-price monetary model should ***not*** be easily dismissed, at least as conceptual starting point.

Nominal Exchange Rate Regimes

- Today we see sizeable fluctuations in nominal exchange rates.
- *Prima-facie*, this is consistent with the case made by Friedman (1953) and many others that, in a world where prices and wages are somewhat sticky, E should be highly flexible.
- However, governments are not always very fond of seeing their exchange rate fluctuate wildly – the so-called "fear of floating" (Calvo and Reinhart, 2002).
- Indeed, going back in history, there were long periods in which most exchange rates were virtually fixed.

Nominal Exchange Rate Regimes

- For example, between 1879 and 1913, advanced countries and many emerging markets then pegged their monies to gold.

- This was the so-called classical gold standard era.

- World War I disturbed this worldwide monetary arrangement but soon after, in 1920s, many countries re-pegged their currency to gold (Brazil being one).

- But then the US stock market collapsed and the great depression of the 1930s made if difficult to stick to gold.

Nominal Exchange Rate Regimes

- Then, from the early 1930s, a wave of devaluations swept the world economy.
- These devaluations and capital controls were responses to capital flowing out of countries and their attempt to restore domestic equilibrium (i.e. combat employment losses) and external equilibrium (trade and current account deficits)
- From 1933 until the circa 1950, the world was mostly on non-pegged exchange rate regimes and capital flows were never as free as pre-1930.

Nominal Exchange Rate Regimes

- Then starting in the early 1950s to 1971, the Bretton-Woods system introduced a system of pegged exchange rates (with temporary re-alignment clauses) and again with capital account restrictions (in addition to generally higher trade tariffs than today).

- From the de-linking of dollar-gold parity in 1971, the world moved again to a (mostly) floating or semi-floating regime.

- Yet some countries maintained to stick, at least for some sub-periods to hard and soft peg exchange rate regimes.

Nominal Exchange Rate Regimes

- Gradually, more and more countries adopted less rigid exchange rate regimes (largely in the context of inflation targeting) and reduced regulatory controls on their financial account.

- That's largely the world of today, but still fixed nominal exchange rates never quite entirely lost their appeal.

- This is clear from the European exchange rate mechanism (ERM) of the 1990s, finally culminating in the adoption of a common currency among core eurozone countries from January 1999.

- Let's now look formally at what requirements a pegged exchange rate regime demands from macroeconomic policy

Nominal Exchange Rate Regimes

Consider our base monetary open economy model with money demand a la Cagan as in (17.18):

$$m_t - e_t = -\eta(E_t e_{t+1} - e_t)$$

If the government to peg the exchange rate at $\bar{e}$, and abstracting from credibility issues in policy implementation, this implies that:

$$m_t - e_t = -\eta(\bar{e} - \bar{e})$$

$$\therefore m_t = \bar{m} = \bar{e} \qquad (7.20)$$

Nominal Exchange Rate Regimes

What (7.19) says is that once you peg the exchange rate, money supply becomes an endogenous variable.

Obviously, a constant money supply is an extreme assumption arising from assuming y, i*, and p* constant and normalized to zero.

Yet, the key point is that, once the government is committed to a policy of fixing the exchange rate, and capital is freely mobile, the government gives up control of the money supply or, equivalently, of setting the domestic interest rate!

Nominal Exchange Rate Regimes

- This dilemma can be seeing clearly in the context of an small open economy that takes i* as given by invoking the UIP of equation (7.2):

$$i_t \simeq i_t * + E_t \Delta e_{t+1} + \zeta_t$$

- Once the government credibly pegs the exchange rate, $E_t \Delta e_{t+1} = 0$. If there are no capital controls and default risk, then $\zeta_t = 0$, so i=i*. Hence, the government surrenders the control of the domestic interest i to the rest of world – typically to countries that issue a world reserve currency like the dollar or the euro.

Nominal Exchange Rate Regimes

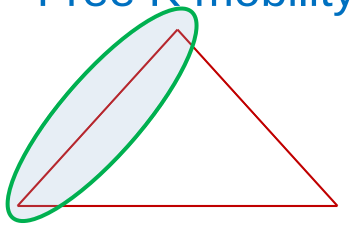
- Yet, from (7.2), it also clear that government can regain some control over i if it can control ζ_t somewhat.
- That is, if the government can put "sand in the wheels" of international capital mobility.
- The government has a variety of ways to impose such "capital controls", notably via differential tax regulations that discriminate foreign investment viz investment by domestic residents.

Mundel's Monetary Policy Trilemma

- Thus, monetary policy faces not a dilemma but a trilemma.

- It can escape from the usual dillemma between fixing e and keep monetary policy sovereignity, but only at the cost of imposing capital controls!

- So, at any point in time policy choices lie at one the sides of the following triangle:



The Mundelian Trillema

- Using historical data, researchers have tested to what extent (if any) such a trillemma has been a bidding constraint on monetary policy of various countries.

- Obstfeld, Shambaugh and Taylor (2005) test the Trillemma by running the following regression:

$$\Delta i_{it} = \alpha + \beta \Delta i^{*}_{it} + u_{it} \qquad (7.21)$$

for various sub-periods, i.e. those when countries floated vs. those when they fixed vs. those when they quasi-fixed.

Mundel's Monetary Policy Trilemma

- If a country has a credible pegged and capital is freely mobile, the trilemma implies that $\beta=1$.

- If $\beta<1$, then the domestic monetary authority has some degree of monetary independence despite the pegged exchange rate and free capital mobility, i.e., the Trilemma is less biding.

- They find $\beta=0.52$ to be the highest for countries under the classical gold standard. For the post-Bretton Woods β=0.46 for pegged and 0.26 for non-pegged.

- For the Bretton-Woods, $\beta=-0.2$!

Mundel's Monetary Policy Trilemma

- One take-home from these results is that the monetary policy trilemma is not that overwhelming as in theory but it is nevertheless strong

- A $\beta=0.52$ indicates that once you peg the nominal exchange rate, your domestic interest is significantly affected by the foreign monetary policy (as measured by i*).

- A $\beta=0.26$ for non-pegged regimes in the post-Bretton woods indicates that once you float the exchange rate you reduce that influence.

- Also consistent with the Trilemma, a $\beta=-0.2$ for the capital control era of Bretton-Woods indicates in turn that capital controls can greatly help in reducing the i-i* link.

Speculative Attack on a Fixed Exchange Rate

- One much documented problem with fixed exchange rate regimes is that it is open to the possibility of a speculative attack on the peg.
- This is specially risky if fundamentals are not sound enough.
- One important element in this vector of fundamentals is, once again, the fiscal stance.
- Krugman (1979) and Flood and Garber (1984) develop a nice model which rationalize how this can happen.

Speculative Attack on a Fixed Exchange Rate

- We have seen that fixing e requires:

$$m_t = \overline{m} = \overline{e} \tag{7.22}$$

- Consider now the central bank balance sheet:

$$M_t = B_{H,t} + \varepsilon B_{F,t} \tag{7.23}$$

- Suppose the government runs G-T=ΔB, so that:

$$\frac{\dot{B}_H}{B_H} = \dot{b}_H = \mu \tag{7.24}$$

Speculative Attack on a Fixed Exchange Rate

- But we have seen that M has to be fixed for e to be fixed.

- So, from the central bank balance sheet, Bf has to adjust:

$$\dot{B}_H = -\bar{\varepsilon}\dot{B}_F \tag{7.25}$$

- Insofar as foreign bonds held by the central bank (usually denominated in reserve currencies like the dollar or the euro) equal FX reserves, then the flip side of central bank financing of fiscal deficits is a loss of reserves.

Speculative Attack on a Fixed Exchange Rate

- Since reserves are finite, if the fiscal deficit persists, the country will eventually run out of reserves.
- Once this happens it is clear that deficits can only be financed my increases in money supply.
- But this is inconsistent with a fixed exchange rate (all else constant).
- So, the exchange rate peg collapses!

Speculative Attack on a Fixed Exchange Rate

- The beauty of the Krugman-Flood-Garber model is to show that the exchange rate collapses **before** FX reserves are wiped out.

- That is, currency crises can take place with falling but yet positive reserves.

- The model also gives us tools to predict the timing of the speculative attack.

Speculative Attack on a Fixed Exchange Rate

- The intuition is subtle: if everyone were to expect the exact t in which Bf=0, then this would be a for sure capital gain so investors fully anticipating it would buy all the FX reserves today!

- That is, in equilibrium, there must some uncertainty for the individual investor on when the jump in e occurs, and this has to be before Bf=0.

- The key is to compute the shadow exchange rate once the attack has taken place.

Speculative Attack on a Fixed Exchange Rate

- The key step to obtain the solution is to compute the shadow exchange rate, i.e., the exchange rate that is consistent with the (fiscal fundamentals) and once all CB reserves are gone so $M=B_H$.

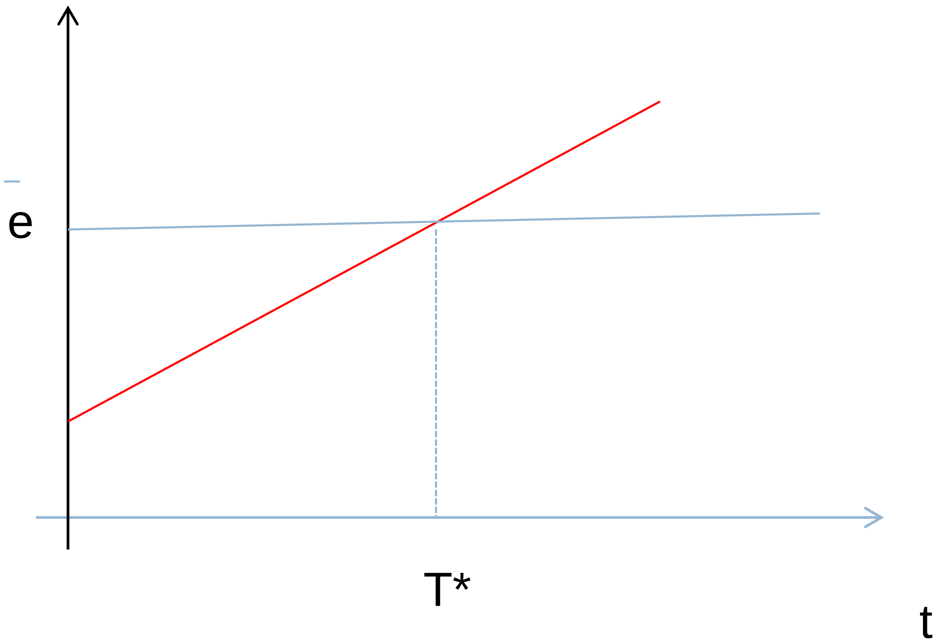
We compute the shadow rate $\tilde{e}_t$ from the Cagan equation:

$$\begin{aligned} &\tilde{e}_t - m_t = \eta(E_t e_{t+1} - e_t) \\ &.: \tilde{e}_t = b_{ht} + \eta\mu \end{aligned} \qquad (7.26)$$

The solution for the model will be the point where $\tilde{e}_t = \overline{e}$, when the attack takes place.

Speculative Attack on a Fixed Exchange Rate

This is the point T*:



Speculative Attack on a Fixed Exchange Rate

Now compute the law of motion of Bh from (7.24):

$$b_{h_t} = b_{H_0} + \mu t \qquad (7.27)$$

Plugging back into (7.26) and solving for T yields:

$$T^* = \frac{\bar{e} - b_{H_0} - \eta\mu}{\mu} \qquad (7.28)$$

Since prior to the attack $\bar{e} = \bar{m} = \ln(B_H + \bar{\varepsilon} B_F)$ then:

$$T^* = \frac{\ln(B_{H_o} + \bar{\varepsilon} B_{F_o}) - b_{H_0} - \eta\mu}{\mu} \qquad (7.29)$$

Speculative Attack on a Fixed Exchange Rate

Equation (7.29) says that the attack will take longer if:

- International reserves are higher to begin with;
- The initial exchange rate ε is more depreciated;
- The semi-elasticity of money demand is lower;
- The fiscal deficit (growth of domestic debt accumulation) is lower.

Speculative Attack on a Fixed Exchange Rate

To show that reserves will still be positive at t<T*, replace B_H for t=T* in (7.23).

$$\bar{\varepsilon} B_{F,t} = \bar{M} - \exp(b_{H_0} + \mu t)$$

$$\therefore \begin{Bmatrix} \bar{\varepsilon} B_{F,t} = \bar{M} - B_{H,o} \exp(\mu t) > 0 \quad \forall t < T^* \\ B_{F,t} = 0 \quad \text{for } t = T^* \end{Bmatrix}$$

So, there is an exponential decay of reserves which however remain positive as long as t<T*, suddenly collapsing to zero exactly when t=T*, but not before.